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Dynamic analysis of Column under Generalized Boundary Condition through Modal Analysis

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Abstract—*The present work focuses on dynamic analysis for column in terms of modal parameters (natural frequencies and mode shape) through Finite Element Method. The column under study include generalized support condition i.e. Fixed-Free, Fixed-Fixed, Fixed-Hinged and Hinged-Hinged. Modal frequencies is also calculated through analytical method using equivalent mass concept. The column analyzed have "I" shape crossection. The mode shape and natural frequencies for all the above mentioned support conditions has been established.*

Keywords: *Finite Element Modeling, Modal Analysis, Column, Mode shape, Natural frequency, ANSYS.*

1. Introduction

Modal analysis is performed to obtain the dynamic properties of systems. Modal analysis is very common technique for determination of dynamic characteristics of any engineering structures. In dynamic analysis influence of acceleration has to be taken in consideration. By this method natural frequencies, mode shapes even damping factor, in other words dynamic response of structures or fluid (confined in a space) can be determined easily.

Column are the structural member commonly used to transfer compressive load. Columns are generally used in one of the four generalized support conditions i.e. Fixed-Free, Fixed-Fixed, Fixed-Hinged and Hinged-Hinged. Street light poles can be considered as the fixed free type column which are subjected to excitation force by wind. Column used in bridges can be fixed-hinged or fixed-fixed type.

The literature survey shows that majority of the work done till date are based on the modal analysis of beams trough different method. Ankit Gautam et.al [1] have analyzed rectangular crossection beam using ANSYS software and validated it through analytical results. Pavol Lengvarsky et.al [2] have presented the steps for the simulation on ANSYS and Solid Works for modal analysis of titan cantilever beam. It was concluded that first five natural frequency and mode shape obtained was identical for ANSYS and Solid Works. Kumar Pankaj [3] studied the modal behavior of beam which include cantilever, fixed-free and fixed-fixed support conditions, using theoretical analysis, simulation in ANSYS and experiment using FFT analyzer. Simulation result deviate less as compared to the experimental value from the theoretical result. Vijaykumar et.al [4] have performed the analysis of "I" section cantilever beam made of tungsten for mode shape and modal frequency, they have performed the simulation by block Lanczos method. The "I" section beam is designed in the ANSYS and solid works. It was observed that the modal frequency which is obtained by ANSYS is same as that obtained by Solid works. Sampath S S [5] have analyzed cantilever beam with T section using ANSYS, they have taken in account all the degree of freedom at free end. Mode shape of the beam at various frequencies was obtained. Chao Ming Ching and Slamet Widodo [6] performed experimental modal analysis for steel cantilever beam using ME'scopeVES modal analysis software. Natural frequency was obtained experimentally. Kumar Vipin[7] have obtained the modal frequency for T and I crossection cantilever beam made of different material. These cantilever beam were designed and analyzed in ANSYS. It was observed that deflection in "T" section is more than "I" section

Bassiouni AS [8] presented a laminated composite beam FE model for the investigation of modal frequency and mode shape. All the lamina of the FE model have equal lateral displacement at a particular crossection. In addition to this every lamina can rotate differently from the other lamina. Mathematical model developed in this case gives better result compared to the experimental values. Kisa [9] demonstrated a method to detect the presence and nature of the cracks by analyzing the variation in mode shapes, and decrease in the natural frequency. Using this method it is possible to analyze the contact which occurs due to closing of crack. Banerjee [10] have developed true dynamic stiffness matrix by considering rotatory inertia, axial force and shear deformation for investigation of free vibrating axially laminated composite beams. The proposed method have application for the analysis of the composite blades of Helicopter and wings. Behzad et.al [11] have presented the method to find the dimension of the cantilever beam to have minimum vibration with the help of modal analysis. By using classical beam theory J.C. Maltbaek, [12] calculated the natural frequencies and mode shapes of the radial drilling machine structure. Farooq and B. Feeny"s [13] have presented a unique method in which modal frequency and mode shapes of the continuous vibrating structure can be obtained. The result obtained by the method proposed are in agreement with the theoretical result. S. Mahalingam [14] demonstrated that due to change in the position of support to the beams modal parameter also changes. A. Cusano et.al [15] not used conventional accelerometer for experimental modal analysis is rather they have used Bragg grating sensors and results were verified by the result obtained by simulation.

Literature survey shows that majority of work has been done on the beams. So in the present work modal parameters (natural frequencies and mode shape) of columns under generalized support condition i.e. Fixed-Free, Fixed-Fixed, Fixed-Hinged and Hinged-Hinged are calculated and compared. In section 2 modeling of the column is done were appropriate dimension of column have been chosen. In section 3 theoretical analysis of the column is done and modal frequency for each support condition is obtained. In section 4 modal frequency is obtained by Finite element analysis of the column. Section 5 and 6 present the results and discussion and the concluding remarks respectively.

2. Design and modeling of Column

The current study involve geometric modeling of column using ANSYS, dimension of column is as per IS 808: 1989, SC 100. The material which is assumed is structural steel with the density 7800 Kg/m^3. The length of the column is assumed to be 5 m. The column will be modeled in ANSYS and will be analyzed under four different support conditions fixed-free, fixed-fixed, fixed-hinged and hinged-hinged.

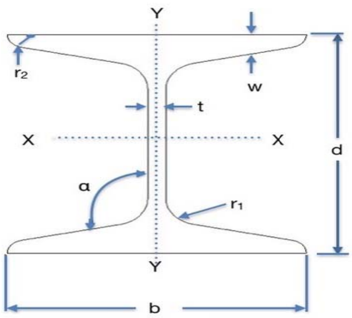


Fig 1: SC 100 Crossection

where, B (flange width of beam, column or channel sections) = 100 mm, D (depth of column) = 100mm, R1 (radius at fillet or root) = 12mm, R2 (radius at toe) = 6mm, α (Flange slope) = 98 degree, t (thickness of web of column) = 6 mm, T (thickness of flange of column) 100 mm, L (Length of column) = 5000mm, m(Mass per unit length) = 20kg/m, I_x (Second moment of area (about x axis) = 436 cm^4, I_y (Second moment of area about y axis) = 136 cm^4, E(Young's modulus) = 200Gpa

3. Theoretical analysis

The general equation of vibration is given by [5]

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \quad \text{.... (1)}$$

Where, [M] = mass matrix of the element; [K] = stiffness matrix; [C] = damping coefficient matrix

The natural frequency of column can also be determined by considering equivalent mass (m_e) [17].

Case-1: **Fixed-free**, equivalent mass is given by the expression (2) and mode-1 frequency for fixed free case is given by the expression (3)

$$m_e = \frac{mL}{4} \quad \text{...(2)}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{3EI_y}{m_e L^3}} \quad \text{...(3)}$$

Above expression gives mode-1 frequency for fixed free case equal to 2.573 Hz. Now for mode-2 frequency I_y is replaced by I_x in equation (3), which gives frequency equal to 4.604 Hz.

For mode-3 vibration displacement of tip of 0.217L length will be maximum [18], the frequency is given by equation (4)

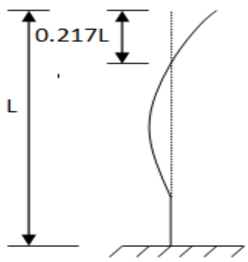


Fig 2: Mode shape of fixed free column for mode 3

$$f = \frac{1}{2\pi}\sqrt{\frac{485.481 EI_y}{mL^4}} \quad \text{...(4)}$$

Above expression gives mode-3 frequency for fixed free case equal to 16.3582 Hz. Similarly for mode-5 I_y is replaced by I_x in equation (4), which we gives frequency equal to 29.2893 Hz. It must be noted here that mode-4 frequency involve torsional vibration which is complex to calculate analytically.

Case-2: Fixed-fixed, equivalent mass is given by the expression (5) and mode-1 frequency for fixed-fixed case is given by expression (6)

$$m_e = \frac{16mL}{35} \quad \ldots (5) \qquad f = \frac{1}{2\pi}\sqrt{\frac{192EI_y}{m_e L^3}} \quad \ldots (6)$$

Above expression gives natural frequency for fixed-fixed case equal to 15.2227 Hz. Now for mode-2 frequency I_y is replaced by I_x in equation (6), which gives frequency equal to 27.2425 Hz. For mode-3 frequency equivalent length of the column is half of the original length as one extra node is formed for this mode shape, which gives frequency equal to 43.0567 Hz. Now for mode-5 frequency I_y is replaced by I_x in equation (6) and taking equivalent length as half of the original length, gives natural frequency equal to 77.09293 Hz. For this case too mode-4 natural frequency cannot be calculated due to torsional mode of vibration.

Case-3: Fixed-hinged, mode-1 frequency is given by the expression (7) [17]

$$f = \frac{1}{2\pi}\sqrt{\frac{237.715EI_y}{mL^4}} \quad \ldots (7)$$

Above expression gives mode-1 frequency for fixed hinged case equal to 11.45244 Hz. . Now for mode-2 frequency I_y is replaced by I_x in equation (7), which gives frequency equal to 20.5056 Hz. For mode-3 frequency equivalent length of the column is half of the original length as one extra node is formed for this mode shape, which gives frequency equal to 45.7865 Hz. Now for mode-5 frequency I_y is replaced by I_x in equation (7) and taking equivalent length as half of the original length, gives natural frequency equal to 81.9807 Hz. For this case too mode-4 natural frequency cannot be calculated due to torsional mode of vibration.

Case-4:Hinged-hinged, mode-1 frequency will be calculated by equivalent mass concept for which me is given by the equation(8) and natural frequency for hinged-hinged column will be given by the equation(9).

$$m_e = \frac{8mL}{15} \quad \ldots (8) \qquad f = \frac{1}{2\pi}\sqrt{\frac{48EI_y}{m_e L^3}} \quad \ldots (9)$$

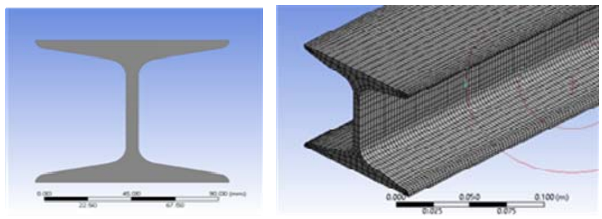
Above expression gives mode-1 frequency for hinged-hinged case equal to 7.046785 Hz. Now for mode-2 frequency I_y is replaced by I_x in equation (7), which gives frequency equal to 12.61725 Hz. For mode-3 frequency equivalent length of the column is half of the original length as one extra node is formed for this mode shape, which gives frequency equal to 19.931 Hz. Now for mode-5 frequency I_y is replaced by I_x in equation (7) and taking equivalent length as half of the original length, gives natural frequency equal to 58.254 Hz. For this case too mode-4 natural frequency cannot be calculated due to torsional mode of vibration.

Table 1: Theoretical natural frequency

Types of support	Fixed-free	Fixed-fixed	Fixed-hinged	Hinged-hinged
Mode-1 frequency(Hz)	2.573	15.2227	11.45244	7.046785
Mode-2 frequency(Hz)	4.6466	27.2425	20.5056	12.6175
Mode-3 frequency(Hz)	16.22466	43.05670	45.7865	19.931
Mode-5 frequency(Hz)	29.066	77.09293	81.9807	35.6869

4. Finite element analysis

ANSYS 18.1 software has been used for finite element analysis of the I-section column. This I-section model of the column was analyzed under four different boundary conditions. After making model another important part is meshing. In meshing an element is divided into finite number of small element and then every small element is solved individually [2]. While meshing small elements which are formed can be 2D or 3D. Sometimes meshing is also referred as "grid generation". By changing the size of the elements formed due to meshing accuracy of the result can also be improved, although large number of the element increases the time of computation by insignificant change in the accuracy. After this step support condition is given to the two end faces of the column before. On the basis of dimension in table 1 the crossection made on the ANSYS is given below

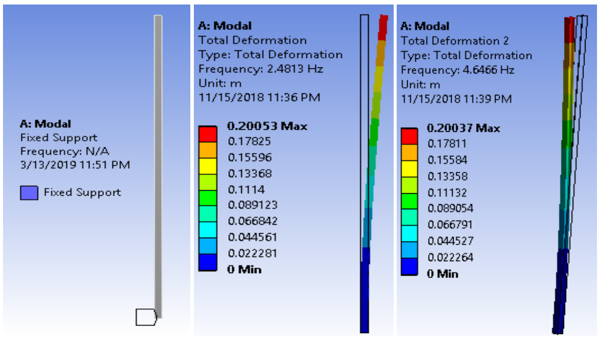


(a) (b)

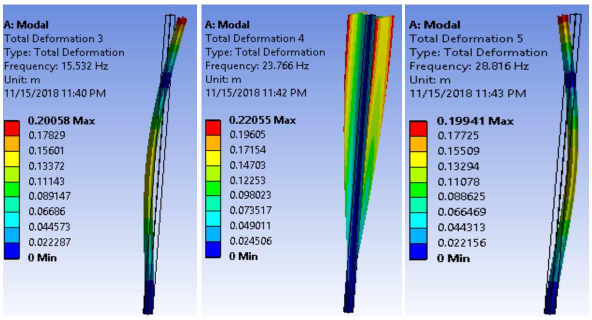
Fig 2: (a) "I" Shape crossection Made on ANSYS (b) Meshing of the Column

4.1 Fixed-free

In this case one end face of the column was fixed (all DOF restricted) while other was kept free. This was achieved by choosing fixed support for one end face due to which all of its degree of freedom were restricted. After solving following five frequencies were obtained. To increase the accuracy of the result mesh size must be decreased, but at the same time errors get accumulated [16]. Although unnecessary decreasing the size of the mesh increases the computation time without significant change in the result. Our result converges with a mesh is having 1551262nodes and 296660 elements. Meshing has average element quality of 0.78738 and average skewness of 0.24961.



(a) (b) (c)

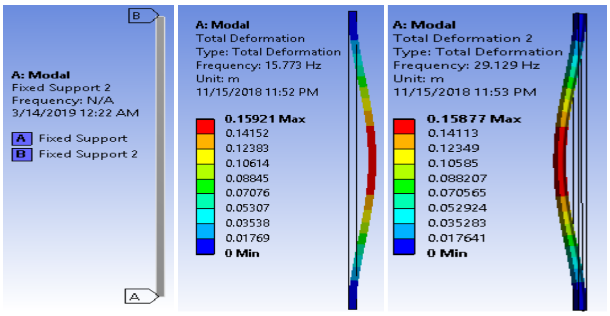


(d) (e) (f)

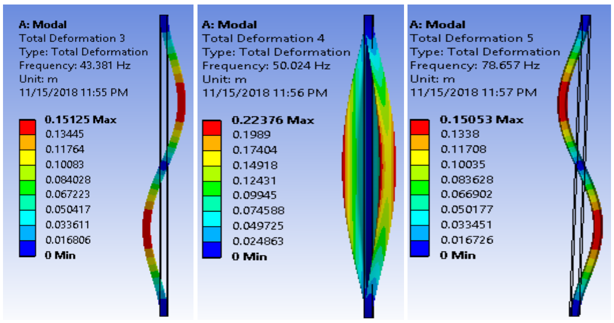
Fig 3; Mode Shape and frequency for fixed-free column, (a)Support condition of on both the end of column (b) Mode shape-1 with frequency 2.4813Hz ,(c) Mode shape-2 with frequency 4.6466Hz, (d) Mode shape-3 with frequency 15.532Hz, (e) Mode shape-4 with frequency 23.766Hz (f) Mode shape-5 with frequency 28.816Hz

4.2 Fixed-fixed

For fixed-fixed support both the end faces of the column were given fixed support condition due to which all DOF of both the faces will be restricted. As in the above case in this case also model updating is done to improve the quality of the meshing. On convergence mesh is having1551262 nodes and 296660 elements. Meshing has average element quality of 0.78738 and average skewness of 0.24961. The natural frequency and mode shape obtained are as follow.



(a) (b) (c)



(d) (e) (f)

Fig 4; Mode Shape and frequency for fixed-fixed column, (a) Support condition of on both the end of column (b) Mode shape-1 with frequency 15.773Hz ,(c) Mode shape-2 with frequency 29.129Hz, (d) Mode shape-3 with frequency 43.381Hz, (e) Mode shape-4 with frequency 50.024Hz (e) Mode shape-5 with frequency 78.657Hz.

4.3 Fixed-hinged

To hinge the column it is necessary to put hole on the web face of the column so that it can be hinged properly.

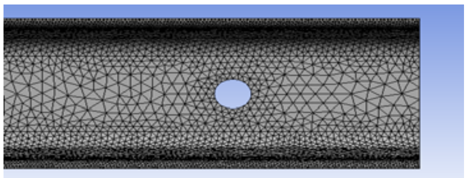
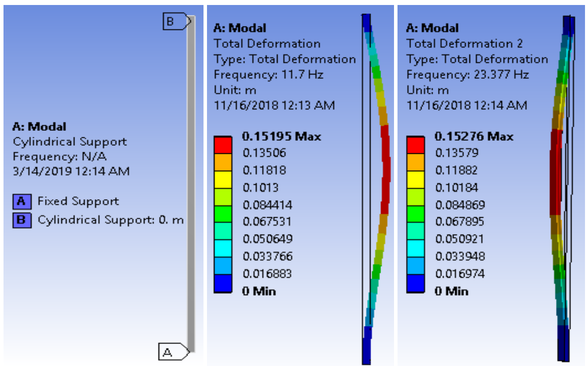
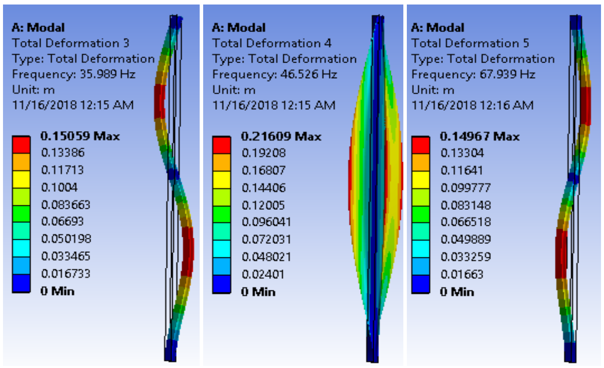


Fig 5: Hole and Meshing around it

After extruding hole cylindrical support is chosen which will restrict all the translational DOF. While the other face is given usual fixed support. On convergence meshing is having 2750939 nodes and 1738938 elements. Meshing has average element quality of 0.82546 and average skewness of 0.25179. In this case number of nodes and elements significantly increases from previous two case due to meshing around hole.



(a) (b) (c)

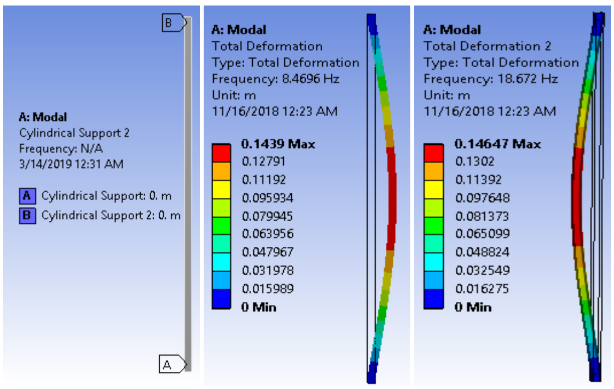


(d) (e) (f)

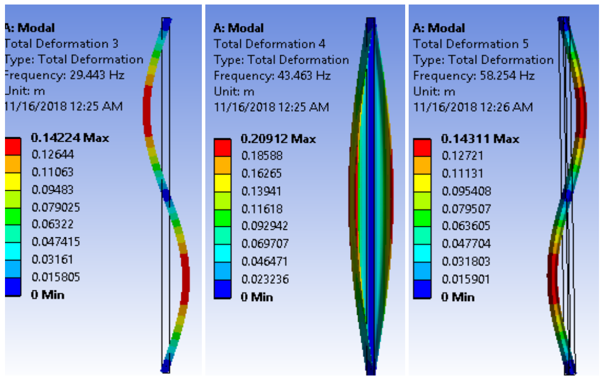
Fig 6; Mode Shape and frequency for fixed-hinged column, (a) Support condition of on both the end of column (b) Mode shape-1 with frequency 11.7Hz ,(c) Mode shape-2 with frequency 23.377Hz, (d) Mode shape-3 with frequency 35.989Hz, (e) Mode shape-4 with frequency 46.526Hz (f) Mode shape-5 with frequency 67.939Hz.

4.4 Hinged-hinged

Similar to above case one more hole is provided on the other side so that column can be hinged on both the side, and cylindrical support is provided for both the holes. On convergence meshing is having 2953324 nodes and 1878882 elements. Meshing has average element quality of 0.8223 and average skewness of 0.2564.



(a) (b) (c)



(d) (e) (f)

Fig 7; Mode Shape and frequency for hinged-hinged column, (a) Support condition of on both the end of column (b) Mode shape-1 with frequency 8.4696Hz ,(c) Mode shape-2 with frequency 18.672Hz, (d) Mode shape-3 with frequency 29.443Hz, (e) Mode shape-4 with frequency 43.463Hz (f) Mode shape-5 with frequency 58.254Hz.

5. Result and discussion

First five natural frequency and mode shape of column for all the four support condition were obtained from ANSYS. The natural frequency was also calculated analytically. It was observed that highest natural frequency is obtained for fixed-fixed support condition which is 78.657Hz for 5th mode

Table 2. Frequency obtained for each mode for each support condition

	Fixed-free	Fixed-fixed	Fixed-hinged	Hinged-hinged
Mode 1 (Hz)	2.4813	15.773	11.7	8.4696
Mode 2 (Hz)	4.6466	29.129	23.377	18.672
Mode 3 (Hz)	15.532	43.381	35.989	29.443
Mode 4 (Hz)	23.766	50.024	46.526	43.463
Mode 5 (Hz)	28.816	78.657	67.939	58.254

The plot below shows the variation of natural frequency for all the four support conditions.

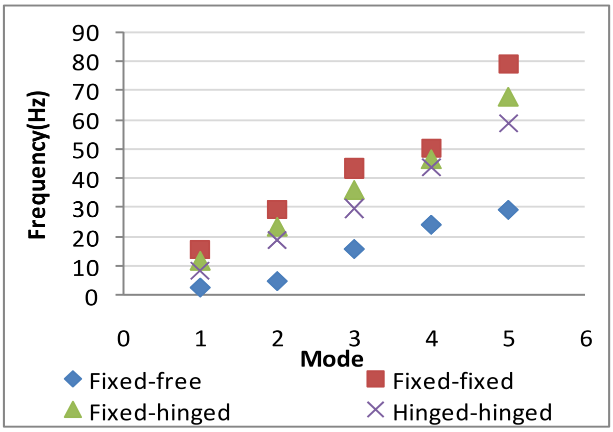


Fig 8: Comparison of modal frequency

Table 3. Maximum displacement for each support condition and for each mode

	Fixed-free	Fixed-fixed	Fixed-hinged	Hinged-hinged
Mode 1 (mm)	200.53	159.21	151.95	143.9
Mode 2 (mm)	200.37	158.77	152.76	146.47
Mode 3 (mm)	200.58	151.25	150.59	142.24
Mode 4 (mm)	220.55	223.76	216.09	209.12
Mode 5 (mm)	199.41	150.53	149.67	143.11

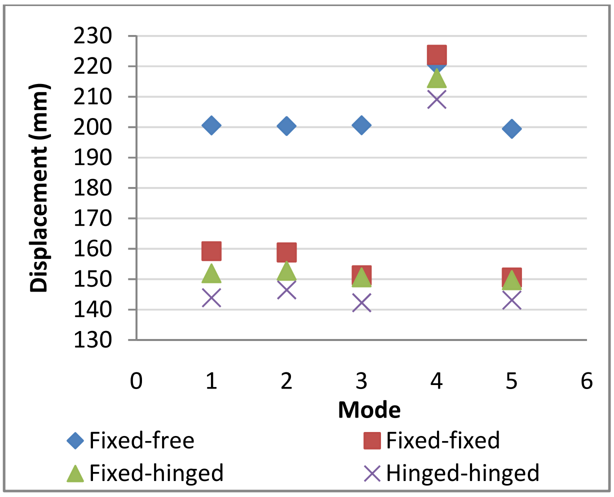


Fig 9: Comparison of max displacement for each mode of all support conditions

From the graph it is clear that 4th natural frequency except for fixed-free condition are very close to each other. Percentage deviation of natural frequency from theoretical value is given in table 4.

Table 4. Percentage deviation of natural frequency from theoretical value.

	Fixed-free	Fixed-fixed	Fixed-hinged	Hinged-hinged
Mode-1	3.56	3.58	2.16	16.8
Mode-2	0.9167	6.476	12.283	32.426
Mode-3	4.269	0.7475	21.398	32.306
Mode-5	0.8634	2.028	17.128	38.7389

From the above table it is observed that maximum percentage deviation is for hinged-hinged support condition, which can be explained by the fact that the column is hinged about I_y but not about I_x due to this asymmetry huge percentage deviation is obtained in the Analytical and simulated result.

6. Conclusion

This paper deals with the estimation of natural frequencies, modal parameter and mode shapes of column for different support condition for different support conditions i.e. Fixed-Free, Fixed-Fixed, Fixed-Hinged and Hinged-Hinged. The natural frequency for each case have been determined analytically. The column has been designed and simulated in ANSYS to obtain the mode shapes and natural frequencies for each. The analytical and simulated natural frequencies are in agreement with each other. The natural frequencies are maximum for the Fixed-Fixed case and minimum for Fixed-Free case. The displacement of column is maximum for Fixed-Free case and minimum for Fixed-Fixed case. This work established the facts that modal parameters of the column are affected by boundary conditions. Therefore the boundary conditions must be chosen depending on the specific application, to ensure the safety and stability of structure.

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